

## APPLICATION OF SOME $C^*$ CONDITIONAL EXPECTATIONS TO STATISTICAL MECHANICS

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The aim of this talk is to present an introductory account of recent works of myself and Moriya, in which some  $C^*$  conditional expectations are defined and used to much improved results than those in the book of Bratteli and Robinson about the equivalence of different characterizations of equilibrium states such as the KMS condition and the variational principle.

A new feature about our conditional expectations is that they are a kind of slice maps for a kind of a product system in which factor algebras of the system are mutually not commuting.

I will focus my talk to the following two points: (1) The association of a unique standard potential with each dynamics, in contrast to the usual approach where a potential is first given (with some artificial convergence condition) and dynamics is defined in terms of the potential. The conditional expectations are used to define the “standard” potential and to characterize the standardness. (2) Conceptually much simplified proof of an energy estimate, in which the conditional expectations are effectively used.

### Reference

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- [3] O.Bratteli and D.W.Robinson, *Operator Algebras and Quantum Statistical Mechanics*, vol.2, second ed. (Springer Verlag, 1997).